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Eliciting Socially Optimal Rankings from Unfair Jurors

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1 Introduction

Conventional wisdom asserts that jurors must be as impartial as possible. Nevertheless, in the real world, this situation seldom arises. Consider the case of a gymnastics competition in which a jury must provide a ranking of the participants. Suppose that there is a true ranking of gymnasts such that the best gymnast is first in the ranking, the second best gymnast is second in the ranking, and so on. Suppose also that the true ranking is known by all jurors, but it is not verifiable.[1] The socially optimal rule is that the contestants be ranked according to the true ranking. In many occasions, however, the jurors are not impartial and, instead of judging the performance of all gymnasts unbiasedly, they want to favor some of the participants over others (for instance, some jurors want to help the gymnasts from some countries and/or to harm the gymnasts from other countries). Another example of this situation is the case in which a group of professors must provide a ranking of students who have applied for scholarships. The professors might not be totally impartial and, for instance, try to favor the students of some of their colleagues and/or to harm the students of others. More examples are rankings of Ph. D. programs, wine tasting, etc.[2]

If the jurors are partial, conventional wisdom affirms that, except by chance, the decisions of such a committee will be biased and will not correspond to socially optimal goals. This belief, however, is not true. There are ways to neutralize the particular interests of partial jurors. Of course, the jurors can be partial in many different ways. Thus, a juror might want to always treat preferentially one or several contestants (his “friends”), or he might want to always prejudice one or several contestants (his “enemies”), or he might want to favor some contestants only when compared with others, etc. Depending on the specific bias of the members of the jury, we may be able to induce them to rank the contestants according to the socially optimal rule or not. If, for example, all jurors wanted to favor a particular contestant over the rest, then it is clear that they would always agree to place that contestant first in the ranking, regardless of what the true ranking was. There

[1] The case in which each juror may observe a different “true ranking” would fall within the area of aggregation of experts’ opinion. Assuming that each juror reveals his “true ranking”, the problem would be to find a compromise between conflicting opinions (see, e.g., Young, 1995).

[2] In many of these cases the only way to find an impartial juror is at the cost of he being ignorant about the true ranking.

are other situations, however, in which, despite the prejudices of the jurors, we can induce them to always provide the true ranking (i.e., the socially optimal rule is implementable). Amorós et al. (2002) provided a first example of this for the case in which each juror wants to favor one (and only one) different contestant over the rest.

In this paper, instead of considering a particular bias for the jurors and analyzing whether the socially optimal rule is implementable in that specific setting, we analyze the problem from a different perspective. We consider a wide class of preferences for the jurors (called "non-impartial") that covers almost any kind of bias that one can imagine, and we study necessary and sufficient conditions on this class of preferences under which the socially optimal rule is implementable. Specifically, we focus on Nash implementation (Nash equilibrium is an appropriate equilibrium concept since in most of the cases, like gymnastics competitions or scholarships, the jurors know each other).

A very weak requirement about the impartiality of a juror over two contestants is to demand that they be an "indifferent pair" for him. This requirement says that, given two rankings where only two consecutive contestants interchange their positions, the juror for whom they are an indifferent pair must prefer the ranking where they are truthfully placed. Of course, the fact that two contestants are an indifferent pair for a juror does not imply that he always judges them unbiasedly (for example, the condition does not determine the preferences of the juror over two rankings where only these two contestants interchange their positions but they are not consecutively placed).

Our first result (Proposition 1) establishes the following necessary condition for Nash implementation of the socially optimal rule: Every pair of contestants must be an indifferent pair for at least one juror. This condition can be interpreted as the minimum degree of impartiality that we must require of the jury in order to guarantee that its decisions will correspond to the socially optimal goals.

We also show that, in order to Nash implement the socially optimal rule, the designer of the mechanism must know who the jurors are for whom each pair of contestants are an indifferent pair (Proposition 2). This means that we cannot rely on the jury to reveal which of its members has among their indifferent pairs to each pair of contestants: Either the designer of the mechanism has this information (and the mechanism depends on it) or the socially optimal rule is not Nash implementable.

Having each possible pair of contestants as an indifferent pair for some juror and knowing who these jurors are, not only are necessary conditions for Nash implementation of the socially optimal rule, but are also sufficient (Theorem 1). This cannot be directly deduced from the well-known result in the theory of Nash implementation (see, e.g., Maskin, 1999) which shows that any rule satisfying monotonicity and No Veto Power is Nash implementable, since the socially optimal rule does not satisfy the later property. Then, in order to prove our result, we present a simple mechanism which Nash implements the socially optimal rule under the condition described above. Components of messages in this mechanism have a straightforward interpretation: Each juror only has to announce a ranking of contestants and a permutation.

There is an established literature dealing with decision making by juries composed of strategic jurors (see, e.g., Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1998; Duggan and Martinelli, 2001). The problem studied in these papers, however, is not directly related to the problem discussed here: The jury must decide whether to convict or acquit a defendant, and the jurors have different opinions about whether the defendant is guilty or innocent. The present paper is also connected with the literature on information transmission between informed experts and an uninformed decision maker (see, e.g., Krishna and Morgan, 2001; Wolinsky, 2002). Nevertheless, the allocation space considered in these papers is much simpler than ours: The decision maker only has to decide whether to undertake a project or not.

The remainder of the paper is organized as follows. Section 2 lays out the general framework and presents the class of non-impartial preferences. Section 3 establishes the necessary and sufficient conditions on this class of preferences under which the socially optimal rule is Nash implementable. This section also presents a simple mechanism that does the work. Finally, Section 4 makes some concluding remarks.

2 Definitions

Let N be a set of $n \geq 3$ contestants. A social alternative, π, is a ranking of the contestants in N. Let Π be the set of all rankings of the contestants in N. For all $\pi \in \Pi$ and $a \in N$, we denote by p_a^π the position of contestant a in ranking π (the smaller the position of a contestant, the better his situation in the ranking is).

The final ranking will be decided by a group of jurors J. We assume that there exists a **true ranking** of the contestants, $\pi_t \in \Pi$, known by all jurors $j \in J$. The **socially optimal rule** is that the contestants should be ranked according to the true ranking. The true ranking, however, is not verifiable.

Jurors' **preferences** over the set of possible rankings may depend on the true ranking. For example, given $N = \{a, b, c\}$, a juror j may prefer ranking (a, b, c) to ranking (a, c, b) if the true ranking was $\pi_t = (a, b, c)$, but prefer ranking (a, c, b) to ranking (a, b, c) if the true ranking was $\pi_t = (a, c, b)$. The notion of preference function captures this idea. Let $\Re$ be the class of preference orderings defined over Π. Each juror $j \in J$ has a **preference function** $R_j : \Pi \longrightarrow \Re$ which associates with each feasible true ranking, $\pi_t \in \Pi$, a preference relation $R_j(\pi_t) \in \Re$. Let $P_j(\pi_t)$ denote the strict part of $R_j(\pi_t)$.

Let 2^N be the collection of all possible pairs of contestants. We say that a pair of contestants $(a, b) \in 2^N$ is an **indifferent pair** for juror j if for any two rankings giving the same position to everyone except for a and b who, moreover, are placed consecutively in both rankings, j always prefers the ranking where a and b are arranged among them according to the true ranking. Let $I_j^{2^N} \subset 2^N$ denote the set of indifferent pairs for juror j. Then, the class of admissible preference functions for j is defined as follows.

Definition 1 *Given the set of indifferent pairs for juror j, $I_j^{2^N}$, the preference function $R_j : \Pi \longrightarrow \Re$ is* ***admissible*** *for j if and only if for all $(a, b) \in I_j^{2^N}$, all $\pi_t \in \Pi$, and all π, $\hat{\pi} \in \Pi$ with:*
(1) $p_a^{\pi} = p_b^{\pi} - 1$,
(2) $p_a^{\hat{\pi}} = p_b^{\hat{\pi}} + 1$,
(3) $p_c^{\pi} = p_c^{\hat{\pi}}$ for all $c \in N \backslash \{a, b\}$, and
(4) $p_a^{\pi_t} < p_b^{\pi_t}$,
we have $\pi P_j(\pi_t) \hat{\pi}$.

If all possible pairs of contestants were an indifferent pair for a juror, the problem of electing the socially optimal ranking would be trivial: We would only need this juror to choose his favorite ranking. We say that juror j is **non-impartial** if there is a pair of contestants which is not an indifferent pair for him (i.e. $I_j^{2^N} \subsetneqq 2^N$). From now on we assume that all jurors are non-impartial. The following example may clarify these notions.

Example 1 *Let* $N = \{a, b, c\}$. *Then* $2^N = \{(a, b), (a, c), (b, c)\}$. *Suppose that set of indifferent pairs for juror* j *is* $I_j^{2^N} = \{(a, b)\}$. *Then, juror* j *is non-impartial since neither* (a, c) *nor* (b, c) *are an indifferent pair for him. Suppose that the true ranking is* $\pi_t = (a, b, c)$. *Consider the rankings* $\pi = (c, a, b)$ *and* $\hat{\pi} = (c, b, a)$. *The only two contestants who change their positions between* π *and* $\hat{\pi}$ *are* a *and* b*, and they are placed consecutively in both rankings. Moreover, in ranking* π*, contestants* a *and* b *are placed among them according to their relative positions in* π_t *(i.e.,* a *goes before* b*). Then, the fact that* (a, b) *is an indifferent pair for* j *implies that any admissible preference function of juror* j *is such that* $\pi P_j(\pi_t)\hat{\pi}$. *Similarly, in case that the true ranking was* $\hat{\pi}_t = (b, c, a)$*, we would have that any admissible preference function of juror* j *is such that* $\hat{\pi} P_j(\hat{\pi}_t)\pi$*, since in* $\hat{\pi}_t$ *contestant* b *goes before contestant* a.[3]

Since $I_j^{2^N} \neq 2^N$, there always exist some $\pi_t \in \Pi$ and some π, $\hat{\pi} \in \Pi$ where only two consecutive contestants change their relative positions such that, when comparing π with $\hat{\pi}$, juror j does not strictly prefer that ranking where these two contestants are truthfully placed. This is the reason why we say that juror j is non-impartial.

Non-impartialness allows for the possibility that the jurors be biased in many different ways. For example, a juror may have several "friends" and/or several "enemies" among the contestants.

Example 2 *Suppose that each juror classifies all contestants in three groups, friends, enemies, and indifferent, so that:*

(1) A juror always prefers his friends to be placed in some positions as low as possible, and his enemies to be placed in some positions as high as possible.

(2) Given some fixed positions for his friends and his enemies, a juror prefers the rest of contestants to be arranged as close as possible to the true ranking.

In our terms, Condition (1) implies that there is no indifferent pair for a juror composed of one friend and one enemy, one friend and one indifferent, or one enemy and one indifferent. Condition (2) implies that any two

[3] However, the fact that (a, b) is an indifferent pair for juror j does not determine whether $(a, c, b)R_j(\hat{\pi}_t)(b, c, a)$ or $(b, c, a)R_j(\hat{\pi}_t)(a, c, b)$. Note also that our assumptions do not determine whether $(a, c, b)R_j(\hat{\pi}_t)(a, b, c)$ or $(a, b, c)R_j(\hat{\pi}_t)(a, c, b)$, since (b, c) is not an indifferent pair for juror j.

contestants included in the set of indifferent are an indifferent pair for the juror. Moreover, an indifferent pair for a juror could be composed of two friends or of two enemies (i.e., juror j may want to favor contestants a and b over the rest and, at the same time, he may want these two contestants to be arranged among them according to the truth, at least when their positions in the ranking are consecutive).

To elicit the socially optimal ranking we must rely on announcements made by jurors. This is the idea behind the concept of a mechanism. Formally, a **mechanism** is a pair $\Gamma = (M, g)$, where $M = \times_{j \in J} M_j$, M_j is the set of possible messages for juror j, and $g : M \to \Pi$ is the outcome function.

A **state of the world** is a pair (R, π_t), where $R = (R_j)_{j \in J}$ is a profile of admissible preference functions and π_t is the true ranking observed by all jurors. Let S be the set of admissible states of the world.

Given a mechanism and a state of the world, the jurors must decide the messages that they announce. We suppose that the jurors know each other (which makes sense in most cases, like gymnastics competitions, scholarships, etc.) and therefore they take their decisions according to the Nash equilibrium concept. The message profile $m \in M$ is a **Nash equilibrium** of mechanism $\Gamma = (M, g)$ when the state of the world is $(R, \pi_t) \in S$ if $g(m) R_j(\pi_t)\ g(\hat{m}_j, m_{-j})$ for all $j \in J$ and $\hat{m}_j \in M_j$. Let $N(\Gamma, R, \pi_t)$ denote the set of Nash equilibria of Γ when the state of the world is (R, π_t).

Our aim is to design mechanisms such that in equilibrium the contestants are ordered according to the true ranking. We call this notion Nash implementation of the socially optimal rule.

Definition 2 *The mechanism $\Gamma = (M, g)$ **Nash implements** the socially optimal rule when, for all $(R, \pi_t) \in S$:*

(1) There exists $m \in N(\Gamma, R, \pi_t)$ such that $g(m) = \pi_t$.

(2) If $m \in M$ is such that $g(m) \neq \pi_t$, then $m \notin N(\Gamma, R, \pi_t)$.

If such a mechanism exists then the socially optimal rule is Nash implementable.

3 Results

We first study some necessary conditions on the composition of the jury for Nash implementation of the socially optimal rule. Our first result shows that if the socially optimal rule is Nash implementable then any pair of contestants must be an indifferent pair for some juror.[4]

Proposition 1 *If the socially optimal rule is Nash implementable, then every pair of contestants must be an indifferent pair for at least one juror.*

The Appendix contains the proof of Proposition 1. The idea of the proof is simple. If there is a pair of contestants that is not an indifferent pair for any juror, then the preferences of the jurors (and therefore the ranking chosen by them) might not change with the true ranking.

Our next result shows that in order to Nash implement the socially optimal rule we not only need to have each possible pair of contestants in the set of indifferent pairs for some juror, but we also need to know who the jurors are that satisfy this weak requirement of impartiality for each pair of contestants (i.e., we cannot rely on the jurors to reveal that information when they play the mechanism).

Proposition 2 *If the socially optimal rule is Nash implementable, then the designer of the mechanism must know who the jurors are that have among their indifferent pairs to each pair of contestants.*

The proof of this result appears in the Appendix. Its intuition is as follows. If the designer of the mechanism does not know whether (a, b) is an indifferent pair for juror 1 or for juror 2, then he must design a mechanism which works for both situations, and this is not possible. Technically, if the designer of the mechanism does not know the sets of indifferent pairs of the jurors, we should extend the notion of state of the world so that the sets $(I_j^{2^N})_{j \in J}$ could be different in different states of the world. This enlargement of the set of admissible states of the world makes Nash implementation of the socially optimal rule impossible.

From Maskin (1999) we know that monotonicity of the socially optimal rule is a necessary condition for its Nash implementation. In our setting this condition says that if a ranking is socially optimal for some state of the world, it must be socially optimal for any other state of the world where that ranking is at least as preferred by all jurors. The proof of Proposition 2 shows

[4] This is the reason for our assumption that there are more than two contestants (if there were only two contestants the problem would be trivial since the two contestants should be an indifferent pair of the same juror).

that, if the designer of the mechanism does not know who the juror is that has among his indifferent to each pair of contestants, the socially optimal rule does not satisfy monotonicity.[5]

From now on we will assume that every pair of contestants is an indifferent pair for one juror at least and that the designer of the mechanism knows the set of indifferent pairs for every juror. The latter assumption implies that the set of indifferent pairs for a juror does not change with the state of the world. Under these conditions the socially optimal rule satisfies monotonicity. To see this, consider two states of the world, (R, π_t) and $(\hat{R}, \hat{\pi}_t)$, where the socially optimal rankings are different (i.e., $\pi_t \neq \hat{\pi}_t$). Then, there exist two contestants, a and b, that are placed consecutively in π_t but that change their relative positions in $\hat{\pi}_t$ (i.e., $p_a^{\pi_t} = p_b^{\pi_t} - 1$ and $p_a^{\hat{\pi}_t} > p_b^{\hat{\pi}_t}$). From our assumptions, there is a juror j that has among his indifferent pairs to (a, b) in both states of the world, (R, π_t) and $(\hat{R}, \hat{\pi}_t)$. Let π be a ranking where a and b interchange their positions with respect to π_t while the rest of contestants have the same positions as in π_t (i.e., $p_a^{\pi} = p_b^{\pi_t}$, $p_b^{\pi} = p_a^{\pi_t}$, and $p_c^{\pi} = p_c^{\pi_t}$ for all $c \in N \backslash \{a, b\}$). From the definition of indifferent pair we have $\pi_t R_j(\pi_t)\pi$ and $\pi \hat{P}_j(\hat{\pi}_t)\pi_t$, and therefore π_t was not at least as preferred by all jurors when the state of the world changed from (R, π_t) to $(\hat{R}, \hat{\pi}_t)$.

A well-known result in the theory of Nash implementation tells us that, if there are at least three jurors, monotonicity together with an additional requirement called No Veto Power would be sufficient conditions to ensure Nash implementation of the socially optimal rule (see, e.g., Maskin, 1999; Repullo, 1987). No Veto Power states that, if all jurors except possibly one agree on a best ranking in some state of the world, then that ranking must be socially optimal for that state of the world.

Unlike what happens in most economics environments, the socially optimal rule does not satisfy No Veto Power.[6] Therefore, the result invoked above is useless in our setting and the general mechanisms proposed in the

[5]In the example proposed in the proof of Proosition 2 (b, a, c) is the socially optimal ranking for the state of the world (R, π_t), and when the state of the world changes from (R, π_t) to $(\hat{R}, \hat{\pi}_t)$, the ranking (b, a, c) remains at least as preferred by all jurors (i.e., there is no juror j and ranking π such that $(b, a, c)R_j(\pi_t)\pi$ and $\pi\hat{P}_j(\hat{\pi}_t)(b, a, c)$). However, (b, a, c) is not socially optimal for the state of the world $(\hat{R}, \hat{\pi}_t)$, contradicting monotonicity.

[6]To see this, consider the profile of preference functions R described in the proof of Proposition 2. Note that (a, b, c) is the most preferred ranking for two of the three jurors when the state of the world is $(R, (a, c, b))$ but, obviously, (a, b, c) is not the socially optimal ranking at that state of the world.

proofs of that result (see, e.g., Maskin, 1999; Repullo, 1987; Saijo, 1988) do not Nash implement the socially optimal rule.

Fortunately, it does not imply that the socially optimal rule fails to be Nash implementable (No-Veto-Power is not a necessary condition for Nash implementation). Our next result shows that, if there are at least three jurors, the socially optimal rule is Nash implementable under the necessary conditions formulated in Propositions 1 and 2 and that, therefore, these conditions are also sufficient.

Theorem 1 *Suppose that (1) there are at least three jurors, (2) every pair of contestants is an indifferent pair for one juror at least, and (3) the designer of the mechanism knows who the juror is that has among his indifferent pairs to each pair of contestants. Then the socially optimal rule is Nash implementable.*

The proof of Theorem 1 is in the Appendix. There, we provide a simple mechanism that Nash implements the socially optimal rule under our assumptions. The mechanism can be informally described as follows. Each juror proposes a ranking and a permutation. If all jurors agree on a ranking π, then this ranking is chosen. If all jurors except one agree on a ranking π, then the ranking of the deviator, π_k, is chosen only if there is a sequence of rankings from π to π_k such that the only difference between each ranking and its predecessor in the sequence is that two contestants, which are an indifferent pair for the deviator k and that are placed consecutively, interchange their positions (moreover, there must not be two contestants who interchange their positions twice in that sequence).[7] Finally, if more than two jurors disagree on the ranking, then the ranking chosen is the result of consecutively applying the announced permutations over the ranking proposed by one of the jurors (say juror 1).

Our mechanism bears some resemblance to the divide-and-permute mechanism proposed by Thomson (2005) to implement the envy-free rule in problems of fair division.[8] The sets of messages are simple and, in contrast to general mechanisms that achieve implementation when it is possible, they do not include whole preference profiles for several jurors or the use of “integer

[7] Note that we need to know who the indifferent pairs are for each juror.

[8] There are, however, several differences between the two mechanisms. For example, in divide-and-permute only two agents propose an alternative, while here all jurors propose a ranking.

games” (the use of such message spaces has been criticized in the literature; see, e.g., Jackson, 1992).

4 Conclusion

We have studied the problem of eliciting the socially optimal ranking of contestants from a jury whose members may be biased. We have established the following necessary conditions: (1) For each pair of contestants there must be at least one juror who prefers these two contestants to be arranged among them according to the truth if their positions in the ranking are consecutive, and (2) the designer of the mechanism must know who the jurors are that satisfy the previous requirement of impartiality for each pair of contestants. Furthermore, we have shown that these two conditions are also sufficient for eliciting the socially optimal ranking from the jury if it has three members at least. In order to prove the last result we have proposed a simple mechanism that Nash implements the socially optimal rule.

APPENDIX

PROOF OF PROPOSITION 1:

Suppose that there is some $(a, b) \in 2^N$ such that, for all $j \in J$, $(a, b) \notin I_j^{2^N}$. Suppose by contradiction that there exists a mechanism $\Gamma = (M, g)$ that Nash implements the socially optimal rule. Let $\pi_t, \hat{\pi}_t \in \Pi$ be such that (1) $p_a^{\pi_t} = p_b^{\pi_t} - 1$, (2) $p_a^{\hat{\pi}_t} = p_b^{\hat{\pi}_t} + 1$, and (3) $p_c^{\pi_t} = p_c^{\hat{\pi}_t}$ for all $c \in N \backslash \{a, b\}$. Then, we can always find some profile of admissible preference functions, $R = (R_j)_{j \in J}$, such that $R_j(\pi_t) = R_j(\hat{\pi}_t)$ for all $j \in J$. Therefore, $N(\Gamma, R, \pi_t) = N(\Gamma, R, \hat{\pi}_t)$. Since Γ Nash implements the socially optimal rule, there exists $m \in N(\Gamma, R, \pi_t)$ such that $g(m) = \pi_t$. Then $m \in N(\Gamma, R, \hat{\pi}_t)$ and $g(m) \neq \hat{\pi}_t$, which contradicts that Γ Nash implements the socially optimal rule.

PROOF OF PROPOSITION 2:

Let $N = \{a, b, c\}$ and $J = \{1, 2, 3\}$. Suppose that, although the designer of the mechanism knows that all pair of contestants are an indifferent pair for some juror, he does not know who these jurors are. Consider the two profiles of preference functions, R, $\hat{R}$, described in Tables I and II (higher rankings in the table are strictly preferred to lower rankings). Note that R is a profile of admissible preference functions for the case in which the jurors' sets of indifferent pairs are $I_1^{2^N} = \{(a, b)\}$, $I_2^{2^N} = \{(a, c)\}$, and $I_3^{2^N} = \{(b, c)\}$. Similarly, $\hat{R}$ is a profile of admissible preference functions for the case in which the jurors' sets of indifferent pairs are $\hat{I}_1^{2^N} = \{(a, c)\}$, $\hat{I}_2^{2^N} = \{(a, b)\}$, and $\hat{I}_3^{2^N} = \{(b, c)\}$. Suppose that there exists a mechanism $\Gamma = (M, g)$ that Nash implements the socially optimal rule. Since the designer of the mechanism does not know who the jurors are that have among their indifferent pairs to each pair of contestants, he does not know whether the profile of preference functions is R or $\hat{R}$, and therefore the same mechanism Γ must work for both profiles of preference functions (i.e., both, R and $\hat{R}$ are admissible). Let $\pi_t = (b, a, c)$ and $\hat{\pi}_t = (a, b, c)$. From Point (1) of the definition of Nash implementation we know that there is $m \in N(\Gamma, R, \pi_t)$ such that $g(m) = \pi_t$. Moreover, from Point (2) of the same definition, we have $m \notin N(\Gamma, \hat{R}, \hat{\pi}_t)$. Note that $g(m)$ is the most preferred ranking for jurors 1 and 3 when the state of the world is $(\hat{R}, \hat{\pi}_t)$. Furthermore, (a, b, c) is the only ranking which is strictly preferred to $g(m)$ for juror 2 when the state of the world is $(\hat{R}, \hat{\pi}_t)$. Therefore, there must be some message for juror 2, $\hat{m}_2 \in M_2$, such that $g(m_1, \hat{m}_2, m_3) = (a, b, c)$ (otherwise m would be a Nash equilibrium of Γ when the state of the world is $(\hat{R}, \hat{\pi}_t)$). Since $g(m_1, \hat{m}_2, m_3) P_2(\pi_t) g(m)$, this contradicts that $m \in N(\Gamma, R, \pi_t)$.

Juror 1		Juror 2		Juror 3	
$R_1(c,a,b)$	$R_1(c,b,a)$	$R_2(b,a,c)$	$R_2(b,c,a)$	$R_3(a,b,c)$	$R_3(a,c,b)$
$R_1(a,c,b)$	$R_1(b,c,a)$	$R_2(a,b,c)$	$R_2(c,b,a)$	$R_3(b,a,c)$	$R_3(c,a,b)$
$R_1(a,b,c)$	$R_1(b,a,c)$	$R_2(a,c,b)$	$R_2(c,a,b)$	$R_3(b,c,a)$	$R_3(c,b,a)$
(a,b,c)	(b,a,c)	(a,b,c)	(b,c,a)	(b,a,c)	(c,a,b)
(b,a,c)	(a,b,c)	(b,a,c)	(b,a,c)	(a,b,c)	(a,c,b)
(a,c,b)	(b,c,a)	(b,c,a)	(c,b,a)	(b,c,a)	(c,b,a)
(b,c,a)	(a,c,b)	(c,b,a)	(a,b,c)	(c,a,b)	(b,a,c)
(c,a,b)	(c,b,a)	(a,c,b)	(c,a,b)	(a,c,b)	(a,b,c)
(c,b,a)	(c,a,b)	(c,a,b)	(a,c,b)	(c,b,a)	(b,c,a)

Table I

Juror 1		Juror 2		Juror 3	
$\hat{R}_1(b,a,c)$	$\hat{R}_1(b,c,a)$	$\hat{R}_2(c,a,b)$	$\hat{R}_2(c,b,a)$	$\hat{R}_3(a,b,c)$	$\hat{R}_3(a,c,b)$
$\hat{R}_1(a,b,c)$	$\hat{R}_1(c,b,a)$	$\hat{R}_2(a,c,b)$	$\hat{R}_2(b,c,a)$	$\hat{R}_3(b,a,c)$	$\hat{R}_3(c,a,b)$
$\hat{R}_1(a,c,b)$	$\hat{R}_1(c,a,b)$	$\hat{R}_2(a,b,c)$	$\hat{R}_2(b,a,c)$	$\hat{R}_3(b,c,a)$	$\hat{R}_3(c,b,a)$
(b,a,c)	(b,c,a)	(a,b,c)	(b,a,c)	(b,a,c)	(c,a,b)
(b,c,a)	(b,a,c)	(b,a,c)	(a,b,c)	(a,b,c)	(a,c,b)
(a,b,c)	(c,b,a)	(a,c,b)	(b,c,a)	(b,c,a)	(c,b,a)
(c,b,a)	(a,b,c)	(b,c,a)	(a,c,b)	(c,a,b)	(b,a,c)
(a,c,b)	(c,a,b)	(c,a,b)	(c,b,a)	(a,c,b)	(a,b,c)
(c,a,b)	(a,c,b)	(c,b,a)	(c,a,b)	(c,b,a)	(b,c,a)

Table II

PROOF OF THEOREM 1:

Let Θ^n be the class of permutations on N and θ_0 be the identity permutation. Let $\Gamma = (M, g)$ be the following mechanism. For all juror $j \in J$ the message space is $M_j = \Pi \times \Theta^n$. For any profile of messages $m = ((\pi_j, \theta_j))_{j \in J}$, $g(m)$ is as follows:

Rule 1. If $\pi_j = \pi$ for all $j \in J$, then $g(m) = \pi$.

Rule 2. If there is some $k \in J$ such that $\pi_j = \pi$ for all $j \neq k$, but $\pi_k \neq \pi$, then

$$g(m) = \begin{cases} \pi_k; \text{ if there is a sequence of rankings, } \pi^1, ..., \pi^s, \text{ such that:} \\ \quad (1)\ \pi^1 = \pi, \\ \quad (2)\ \pi^s = \pi_k, \text{ and} \\ \quad (3) \text{ for each } q \in \{2, .., s\} \text{ there is } (a^q, b^q) \in I_k^{2^N} \text{ with:} \\ \qquad (3.1)\ p_{a^q}^{\pi^{q-1}} = p_{b^q}^{\pi^{q-1}} - 1, \\ \qquad (3.2)\ p_{a^q}^{\pi^q} = p_{b^q}^{\pi^q} + 1, \\ \qquad (3.3)\ p_c^{\pi^{q-1}} = p_c^{\pi^q} \text{ for all } c \in N \backslash \{a, b\}, \text{ and} \\ \qquad (3.4)\ (a^q, b^q) \neq (a^r, b^r) \text{ for all } r \neq q \\ \pi; \text{ otherwise} \end{cases} \tag{1}$$

Rule 3. In all other cases, let $g(m) = \theta_n \circ \theta_{n-1} \circ ... \circ \theta_1(\pi_1)$.

Step 1: For all $(R, \pi_t) \in S$ there exists $m \in N(\Gamma, R, \pi_t)$ such that $g(m) = \pi_t$.

Let $(R, \pi_t) \in S$. Let $m \in M$ be such that $m_j = (\pi_t, \theta_0)$ for all $j \in J$. Then Rule 1 applies and $g(m) = \pi_t$. Moreover, $m \in N(\Gamma, R, \pi_t)$. To see this consider any unilateral deviation by some juror k to $\hat{m}_k = (\hat{\pi}_k, \hat{\theta}_k)$. Then Rule 2 applies to $(\hat{m}_k, m_{-k})$, and therefore $g(\hat{m}_k, m_{-k}) = \hat{\pi}_k \neq \pi_t$ only if there is a sequence of rankings $\pi^1, ..., \pi^s$ as defined in (1). Note that then, the only difference between any two consecutive rankings in the sequence, π^{q-1} and π^q, is that two consecutive contestants that are an indifferent pair for juror k and that are arranged among them according to the true ranking in π^{q-1} interchange their positions.[9] Therefore, $\pi_t = \pi^1 P_k(\pi_t) \pi^2 P_k(\pi_t) ... \pi^{s-1} P_k(\pi_t) \pi^s = \hat{\pi}_k$.

Step 2: For all $(R, \pi_t) \in S$ and all $m \in M$ such that $g(m) \neq \pi_t$, we have $m \notin N(\Gamma, R, \pi_t)$.

Let $(R, \pi_t) \in S$. Let $m \in M$ be such that $g(m) = \pi \neq \pi_t$. Then, there are at least two contestants, a and b, that are placed consecutively in π but that change their relative positions in π_t, i.e., $p_a^\pi = p_b^\pi - 1$ and $p_a^{\pi_t} > p_b^{\pi_t}$. Let $\hat{\pi}$ be the ranking where a and b interchange their positions with respect to π

[9] We know that the two consecutive contestants who interchange their positions when we move from π^{q-1} and π^q are arranged among them according to the true ranking in π^{q-1} because the first ranking of the sequence was π_t and there are not two contestants who interchange their positions twice in the sequence.

(i.e., $p_a^{\hat{\pi}} = p_b^{\pi}$, $p_b^{\hat{\pi}} = p_a^{\pi}$, and $p_c^{\hat{\pi}} = p_c^{\pi}$ for all $c \in N \backslash \{a, b\}$). Let k be the juror for whom (a, b) is an indifferent pair. Then $\hat{\pi} P_k(\pi_t)\pi$.

Case 1. Suppose that Rule 1 applies to m. Then all jurors are announcing the same ranking π. Consider a unilateral deviation by juror k to $\hat{m}_k = (\hat{\pi}, \theta_0)$. By Rule 2 we have $g(\hat{m}_k, m_{-k}) = \hat{\pi} P_k(\pi_t)\pi = g(m)$, and therefore $m \notin N(\Gamma, R, \pi_t)$.

Case 2. Suppose that Rule 2 applies to m.

Subcase 2.1. Suppose that the deviator in m is juror k. If juror k is not announcing ranking π in m (and therefore the rest of jurors are announcing ranking π in m), he can improve by announcing $\hat{m}_k = (\hat{\pi}, \theta_0)$ (as in Case 1). If juror k is announcing ranking π in m, then the rest of jurors must be announcing a ranking $\tilde{\pi} \neq \pi$ such that there is a sequence of rankings from $\tilde{\pi}$ to π where the only difference between each ranking and its predecessor in the sequence is that two contestants who are an indifferent pair for juror k and that are placed consecutively interchange their positions. Obviously, in that case there also exists a sequence of rankings like that which goes from $\tilde{\pi}$ to $\hat{\pi}$ (we just have to add $\hat{\pi}$ to the previous sequence as new last ranking). Then, if juror k unilaterally deviates to $\hat{m}_k = (\hat{\pi}, \theta_0)$, Rule 2 applies and $g(\hat{m}_k, m_{-k}) = \hat{\pi} P_k(\pi_t)\pi = g(m)$, and therefore $m \notin N(\Gamma, R, \pi_t)$.

Subcase 2.2. Suppose that the deviator in m is not juror k. Consider a unilateral deviation by juror k to $\tilde{m}_k = (\tilde{\pi}, \tilde{\theta}_k)$, where $\tilde{\pi}$ is a ranking different from both, the ranking announced by the deviator in m and the ranking announced by the rest of jurors in m, while $\tilde{\theta}_k$ is such that $\theta_n \circ ... \circ \tilde{\theta}_k \circ ... \circ \theta_1(\pi_1) = \hat{\pi}$. By Rule 3 we have $g(\tilde{m}_k, m_{-k}) = \hat{\pi} P_k(\pi_t)\pi = g(m)$, and therefore $m \notin N(\Gamma, R, \pi_t)$.[10]

Case 3. Suppose that Rule 3 applies to m. Then there are at least three jurors announcing different rankings in m. Consider an unilateral deviation by juror k to $\tilde{m}_k = (\pi_k, \tilde{\theta}_k)$, where π_k is the same ranking that he was announcing in m, while $\tilde{\theta}_k$ is such that $\theta_n \circ ... \circ \tilde{\theta}_k \circ ... \circ \theta_1(\pi_1) = \hat{\pi}$. Again, by Rule 3 we have $g(\tilde{m}_k, m_{-k}) = \hat{\pi} P_k(\pi_t)\pi = g(m)$, and therefore $m \notin N(\Gamma, R, \pi_t)$.

[10]Note that in Rule 3 each juror can make any ranking to be chosen by appropriately choosing his permutation, and this, independently of the permutations chosen by the others.

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